
IMC Number Theory

1. [IMC 2011 Q22] You are given that $5^p = 9$, $9^q = 12$, $12^r = 16$, $16^s = 20$ and $20^t = 25$. What is the value of $pqrst$?
 A 1 B 2 C 3 D 4 E 5

2. [IMC 2011 Q24] Given any positive integer n , Paul adds together the distinct factors of n , other than n itself. Which of these numbers can never be Paul's answer?
 A 1 B 3 C 5 D 7 E 9

3. [IMC 2010 Q20] Shabbaz thinks of an integer, n , such that the different between \sqrt{n} and 7 is less than 1. How many different possibilities are there for n ?
 A 13 B 14 C 26 D 27 E 28

4. [IMC 2010 Q24] A new taxi firm needs a memorable phone number. They want a number which has a maximum of two different digits. Their phone number must start with the digit 3 and be six digits long. How many such numbers are possible?
 A 288 B 280 C 279 D 226 E 25

5. [IMC 2009 Q16] How many different positive integers n are there for which n and $n^3 + 3$ are both prime numbers?
 A 0 B 1 C 2 D 3 E infinitely many

6. [IMC 2009 Q18] If p, q are distinct primes less than 7, what is the largest possible value of the highest common factor of $2p^2q$ and $3pq^2$?
 A 60 B 45 C 36 D 20 E 15

7. [IMC 2007 Q18] One of the digits 1 to 9 is put in each unshaded square so that no digit is repeated and the totals of the entries in the rows and columns are as shown.
 What number goes in the starred square?
 A 1 B 3 C 5 D 7 E 9

| | | | | |
|-------|---|----|-------|----|
| | | | Total | |
| | | | | 12 |
| | | | | 7 |
| | | * | | 13 |
| Total | 4 | 16 | 12 | |

8. [IMC 2007 Q19] The following sequence continues indefinitely:
 $27 = 3 \times 3 \times 3$, $207 = 3 \times 3 \times 23$, $2007 = 3 \times 3 \times 223$, $20\,007 = 3 \times 3 \times 2223$, ...
 Which of the following integers is a multiple of 81?
- A 200 007 B 20 000 007 C 2 000 000 007 D 200 000 000 007
 E 20 000 000 000 007
9. [IMC 2007 Q23] As n takes each positive integer value in turn (that is, $n = 1$, $n = 2$, $n = 3$, and so on) how many different values are obtained for the remainder when n^2 is divided by $n + 4$?
- A 1 B 8 C 9 D 16 E Infinitely many
10. [IMC 2006 Q25] Given that $5^j + 6^k + 7^l + 11^m = 2006$ where j , k , l and m are different non-negative integers, what is the value of $j + k + l + m$?
- A 6 B 7 C 8 D 9 E 10
11. [IMC 2005 Q20] One of the following is the largest of nine consecutive positive integers whose sum is a perfect square. Which one is it?
- A 118 B 128 C 138 D 148 E 158
12. [IMC 2004 Q20] What is the largest power of 2 that divides $127^2 - 1$?
- A 2^1 B 2^7 C 2^8 D 2^{63} E 2^{127}
13. [IMC 2004 Q24] If p , q and $p - q$ are all positive integers, which of the following is least?
- A $\frac{q^2}{p^2}$ B $\frac{p^2}{q^2}$ C $\frac{q}{p}$ D $\sqrt{\frac{q}{p}}$ E $\sqrt{\frac{p}{q}}$

Solutions:

1. B
2. C
3. D
4. B
5. B
6. B
7. C
8. E
9. C
10. D
11. D
12. C
13. A